Accessibility Strategies Toolkit for Mathematics

By Amy Brodesky, Caroline Parker, Elizabeth Murray, and Lauren Katzman

> "Equity does not mean that every student should receive identical instruction; instead, it demands that reasonable and appropriate accommodations be made as needed to promote access and attainment for all students."
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> -- *Principles and Standards for School Mathematics* (NCTM, 2000, p.12)

The goal of this document is to provide an organized list of strategies for making mathematics more accessible to students with disabilities. Standards-based mathematics curricula provide opportunities for students to conduct hands-on investigations, use multiple representations, work collaboratively, and communicate mathematical ideas. The lessons are carefully designed to promote student learning of key mathematical concepts, skills, and processes. However, no curriculum can provide all possible learning alternatives to meet all students' learning needs. An activity that draws on the strengths of some students may pose challenges for others. For example, a student with strengths in visual-spatial processing may excel at working with geometric representations and rotating figures on the coordinate plane, while a student with difficulties in this area may struggle with those tasks, preferring verbal descriptions to visual representations.

Students' strengths and needs in the following eight areas have a strong impact on their success with mathematics: conceptual processing, language, visual-spatial processing, organization, memory, attention, psycho-social, and fine-motor skills. For each of these eight areas, we identified common types of tasks in standards-based middle school mathematics curricula, student needs and challenges, and corresponding teaching strategies for promoting accessibility. The goal of accessibility strategies is to provide the support students need to succeed, while maintaining the integrity of the mathematics and pedagogical approach of the curriculum. While these strategies are targeted at improving the learning experience for students with disabilities, many of them are also common teaching strategies that you are probably already using in your classroom. We invite you to add to the list of strategies.

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Conceptual Processing

Standards-based mathematics emphasizes the need for students to build a deep understanding of mathematical concepts. Understanding concepts involves making connections between ideas, facts, and skills and the metacognitive process of reflecting upon and refining that understanding. In middle school, students begin to work with more abstract mathematical concepts, such as variables and linear functions. They use more symbolic representations than in the elementary grades. Students who tend to think concretely may need additional support to help them make the transition from concrete to abstract representations.

Conceptual		
Types of Tasks	**Example Student Difficulties**	**Accessibility Strategies to Consider**
Use or manipulate symbols	• Does not always connect symbols with what they represent • Does not remember the meaning of symbols	• Use manipulatives (such as Algebra Lab Gear) to connect symbols to concrete objects • Post wall charts or provide resource sheets with symbols and meanings
Solve abstract problems	• Does not understand abstract problems easily • Tends to think concretely	• Set up the investigation so that students move from the concrete to the abstract • Make connections to familiar contexts
Visualize and extend patterns	• Has difficulty visualizing and identifying patterns	• Use manipulatives to build and extend patterns • Provide simpler patterns for students
Make generalizations	• Finds it difficult to make generalizations and to write rules • Tends to think concretely	• Provide generalizations for students to test • Have students use multiple representations of situation and then make a generalization
Understand mathematical relationships and make connections	• Thinks of mathematics as disparate parts and doesn't see the connections	• Make explicit connections between current and prior lessons or units • Use concept maps
Learn, represent, and explain new concepts	• Tends to think concretely • Focuses on small parts and does not see big picture • Does not identify key points	• Use hands-on investigations to build understanding • Contrast examples and non-examples of a concept • Provide resource sheets with summary information on complex concepts • Use frequent assessments to identify any gaps in the students' understanding of concepts • Use multiple representations of concepts • Make concept maps • Provide organizers for students to complete • Use concept map software like Inspiration
Apply concepts to new situations	• Sees new problems as unfamiliar • Does not see a connection between new problems and those he or she has already solved	• Help students to see the connections between new problems and prior work
Self monitor understanding and ask clarifying questions	• Lacks a metacognitive awareness of what he/she doesn't understand	• Have students to reflect on their own learning using questions from KWL strategy: "What do I **K**now? What do I **W**ant to learn? What have I **L**earned?"

Language

As part of the Communications Standard (NCTM, 2000), students need to describe strategies, explain their reasoning, justify solutions, and make persuasive arguments, both orally and in writing. They need to learn mathematical vocabulary and use it to express mathematical ideas with precision and clarity. In class and small group discussions, they need to build on the thinking of their classmates and to ask questions to help them understand and clarify another person's strategies.

Language		
Types of Tasks	**Example Student Difficulties**	**Accessibility Strategies to Consider**
Read directions and problems	• Has difficulty decoding words • Reads slowly	• Read aloud • Use a tape recorder (or use taped texts from *Recordings for Blind and Dyslexic*) • Digitize materials and use text-to-speech software, such as eReader and TexEdit
	• Finds comprehension challenging • Tends to misunderstand directions	• Have students highlight key points and identify unnecessary information • Use pre-reading questions to focus their attention
Follow verbal directions	• Has difficulty with the auditory processing of verbal information • Does not understand verbal directions well	• Provide written as well as oral directions • Make handouts of the overhead masters • Have students rephrase directions in their own words • Use an overhead
Write explanations of mathematical thinking	• Takes a long time to get started on writing tasks	• Reword the question as a statement for students to complete • Have students talk about ideas with a partner before writing them down
	• Does not know how to organize ideas	• Use graphic organizers and writing templates, such as paragraph templates • Use the same writing process as Language Arts • Teach organizational strategies • Use outlining software such as Inspiration
	• Gets distracted rather than focusing on the writing task	• Break writing tasks into smaller parts and provide frequent feedback
	• Does not have necessary fine-motor skills for extended writing	• Have the student dictate to a "scribe" • Use a computer or portable keyboard such as Alpha-smart • Have the student record ideas on a tape recorder
Participate in Class Discussions	• Does not express ideas orally with ease	• Prearrange when you will call on the student or use a nonverbal signal
	• Does not listen well to other students' explanations and gets distracted easily	• Reduce the time for whole group discussions. • Break class into small discussion groups and then have groups report back to the whole group
Give Oral Presentations	• Is not comfortable speaking in front of class • Speaks slowly	• Provide an organizer with questions for preparing the talk • Provide practice time

Visual-Spatial Processing

The representation of mathematical ideas is one of the ten standards in the *Principles and Standards for School Mathematics* (NCTM, 2000). Students create and use representations to solve problems and to explore and communicate mathematical concepts in all the strands. For example, in the number and operation strand, students use different visual representations for percents including number lines, fraction circles and bars, base ten blocks, and hundred-grids. In algebra, students extend visual patterns in order to determine a rule, analyze graphical representations of functions, and create mathematical models.

Some students' difficulties with these types of tasks are caused by a breakdown in the processing of visual information while others are the result of physical impairments. In the former case, students may benefit from using color-coding systems to help them focus on key information and from learning explicit strategies for interpreting visual representations. Students who are blind or have visual impairments require accommodations, such as Braille versions of the text, tape recordings of the text (available from *Recordings for the Blind and Dyslexic*), and text-to-speech software.

Visual-Spatial Processing		
Types of Tasks	**Example Student Difficulties**	**Accessibility Strategies to Consider**
Create and interpret visual representations	• Has difficulty representing mathematics concepts visually • Does not connect graphics to the concepts they represent	• Provide handouts of the representations for students to draw on, highlight, measure, and cut out • Provide manipulatives
	• Finds it difficult to visualize and represent a three-dimensional model in two dimensions • Finds it difficult to interpret a two-dimensional representation of a three-dimensional model	• Provide examples of actual 3-D models for students to view or manipulate
Work with tables and graphs	• Has difficulty figuring out how to create tables or graphs or has difficulty physically creating them • Has difficulty reading or interpreting graphs	• Provide templates that address particular needs (for example, larger or partially filled-in tables) • Use scaffolding strategies to help students eventually develop their own templates for tables, charts, and graphs
Read text	• Cannot read standard-size text	• Use larger fonts • Provide oral versions (spoken, taped) of the instructions and text, where appropriate • Use text-to-speech software • Provide Braille version of the text
Read handouts and book pages	• Finds crowded pages distracting	• Reorganize the material into a handout • Make all of the handouts single-sided
	• Has difficulty focusing on the important information • Finds extraneous material distracting	• Have students highlight the key information • Eliminate extraneous page features • Explicitly teach how to find information in the book
Copy or read information displayed on a blackboard, chart, or overhead	• Does not see board well • Does not know where to focus	• Use large font sizes for overhead masters and give copies of the masters as handouts • Seat students close to the blackboard • Reduce the glare from the windows • Use a consistent format for displaying information on the board

Organization

The *Principles and Standards for School Mathematics* emphasize the integral role of problem solving in mathematical learning. The Problem Solving standard states that "students should have frequent opportunities to formulate, grapple with and solve complex problems that require a significant amount of effort." (NCTM, 2000) Solving these complex problems involves several organizational demands: figuring out how to get started; carrying out a sequence of steps; keeping track of the information from prior steps; monitoring one's progress and adjusting the strategies accordingly; and presenting solutions in an organized manner. Students must also organize their time to insure that they do not rush through tasks and make careless errors or spend an excessive amount of time and not complete the task.

Organization		
Types of Tasks	**Example Student Needs**	**Accessibility Strategies to Consider**
Solve multi-step or complex problems	• Has a hard time getting started	• Provide hints or prompts • Teach problem-solving strategies
	• Does not know how to figure out a sequence of steps for solving the problem	• Teach organizational strategies such as breaking the problem into smaller parts • Give frequent feedback
	• Rushes through tasks or spends excessive time	• Teach organizational strategies for using time wisely • Set expectations for how much time students should spend on tasks • Remind students of how much time remains for completing the tasks
	• Does not answer all of the questions or all parts of the investigation	• Explicitly teach about the layout of the book and the question formats • Provide a handout of the questions that students can highlight or underline
Make a table, graph, chart, number-line, spinner, or map	• Gets confused by the multiple steps involved in making a table, graph, and so on.	• Provide Resource Sheets that list the steps involved or provide examples or templates
Collect and record data	• Records data in a disorganized manner that is difficult to analyze • Has difficulty organizing data into tables	• Use table templates for data collection
Find information in prior student work	• Does not organize class notes well and thus has trouble finding the needed information	• Use a notebook organization system and reinforce it with notebook checks (if possible, use the same notebook organization system across subject areas)

Memory

Both long-term memory and short-term memory play essential roles in the learning of mathematics. For example, students use their memories to perform calculations and procedures, identify geometric figures, and create graphs that have all of the necessary parts. Students with long-term memory deficits may not easily store information, such as number facts or the steps of algorithms, in memory, or they may have difficulty retrieving information. Long-term memory difficulties also impact students' abilities to use mathematical vocabulary and to make connections between concepts that they have learned in prior months or years.

Some students may have problems with short-term or working memory. This is the aspect of memory used to keep track of several pieces of information for a brief time, such as keeping track of calculations when solving multi-step problems or performing mental calculations. Short-term memory difficulties also impact students' abilities to remember directions, to follow a teacher presentation, and to build on other students' responses in a class discussion.

Memory		
Types of Tasks	**Example Student Difficulties**	**Accessibility Strategies to Consider**
Use basic arithmetic facts	• Has difficulty memorizing or recalling basic facts • Retrieves incorrect facts	• Allow students to use a number line • Provide a multiplication chart • Ask students to find patterns in the facts • Allow the use of calculators
Carry out algorithms	• Does not remember sequence of steps in an algorithm	• Provide a model of worked-out calculations, highlighting the steps • Teach mnemonic devices • Provide practice problems and examples • Allow the use of calculators
Perform mental calculations	• Cannot keep the steps of a calculation in his or her working memory	• Allow students to use pencil and paper • Have students talk about which operations they would use instead of calculating • Allow the use of calculators
Solve multi-step problems	• Does not have needed information in his or her working memory to solve a problem	• Provide resource sheets • Provide templates or organizers for recording information • Break the problem into smaller chunks • Allow the use of calculators
Use previously-taught skills and concepts	• Does not remember skills and concepts that were taught earlier in the year or in previous years	• Use a notebook organization system to help students find info. in their prior work • Review the needed skills at the beginning of the lesson or in the resource room • Provide resource sheets with cues to remembering the skills
Use math vocabulary	• Has difficulty remembering math vocabulary	• Preview the needed vocabulary prior to the lesson • Have students look up vocabulary words and write the definitions on a resource sheet • Provide resource sheets for the needed vocabulary

Attention

In middle school, the increasing complexity of math content and types of tasks leads to increasing demands on students for extended attention spans. Students have to listen to directions and explanations, participate in class discussions, and work effectively by themselves. They need to complete multi-step investigations and long-term projects, pay attention to details, and complete tests and assessments, often within a limited time frame.

Attention		
Types of Tasks	**Example Student Difficulties**	**Accessibility Strategies to Consider**
Complete long-term projects	• Cannot maintain attention for the details needed to complete the project • Loses track of what needs to be completed	• Provide a project organizer • Schedule frequent check-in points for longer projects
Complete math work accurately	• Makes careless errors because of going too quickly or poor attention to detail	• Encourage or require that students check their own work
Focus on teacher presentations	• Gets distracted easily • Has difficulty listening for long periods of time	• Provide key questions to help students focus • Use visuals • Include student activities and participation
Work in pairs or small groups	• Distracts the group	• Set clear behavioral and academic expectations • Assign group roles, such as recorder
Participate in class discussions	• Distracts the group • Does not listen to other students • Makes irrelevant comments	• Use visuals • Reduce the time for whole class discussions • Break into small groups and have them report back to large group
Work with manipulatives	• Uses manipulatives for activities that are not task-oriented	• Set clear behavioral and academic expectations • Check-in frequently on manipulative use

Psycho-Social

Standards-based mathematics places a strong emphasis on the communication of mathematical ideas through classroom discourse. Students work together in pairs or small groups to carry out mathematical investigations and then share their findings in a whole class discussion. They may give their peers constructive feedback to help them improve a problem solution or project report. Students need confidence to try new mathematical investigations, to persist through frustration, and to share their ideas in public. All these types of tasks involve psycho-social skills.

Some students may misread social cues and thus cause tensions when they are working with peers. They may make inappropriate comments and disrupt class discussions. In these cases, teachers need to select small groups with care and set up structures for collaborative work and participation in discussions. Other students may give up easily on tasks because they are easily frustrated or lack confidence in their math abilities. These students may benefit from getting frequent feedback and from building on prior successes.

Psycho-Social		
Types of Tasks	**Example Student Difficulties**	**Accessibility Strategies to Consider**
Work in pairs or groups	• Finds that peer relationships cause tensions because of weak social skills	• Set clear expectations for student collaboration and individual accountability in small groups • Choose groups with a specific purpose in mind: sometimes to mix skill levels, sometimes to promote particular social interactions
Move through a frustration point	• Strays from the concept being learned because of frustration, and focuses instead on lack of understanding • Gets frustrated easily • Lacks confidence • Fears failure	• Connect a new concept to one with which students have experienced success • Check to make sure students have the necessary prerequisites • Provide additional support time • Provide frequent feedback
Play math games that involve winning and losing	• Focuses too much on competitive aspect • Is overly sensitive to losing	• Minimize the competitive aspects of the game by presenting it as an experiment • Make careful choices in assigning partners and teams
Give and receive constructive feedback	• Does not know how to give constructive feedback to peers • Is overly sensitive to negative feedback	• Model giving and receiving constructive feedback • Use a structured feedback process and provide handouts for the students to fill out

Fine-Motor

Fine-motor skills require precise, coordinated movements of the fingers and hands. These skills are needed to carry out a variety of mathematical tasks including performing calculations, writing explanations, making tables and graphs, using manipulatives, drawing representations, cutting out shapes, and building scale models. Students with fine-motor problems have difficulty with tasks such as aligning numbers, plotting points on graphs, and drawing straight lines. They tend to work slowly and their final products may be illegible or lack the necessary precision. These students may benefit from having additional time for tasks and from using prepared templates that minimize the amount of drawing or cutting needed. Some students with physical disabilities may need alternative means for approaching fine-motor tasks, such as using software graphing programs, using software versions of manipulatives, dictating explanations into a tape recorder, or working with a partner who is responsible for writing and drawing.

Fine-Motor		
Types of Tasks	**Example Student Difficulties**	**Accessibility Strategies to Consider**
Write explanations, solutions, or reports	• Does not write well by hand, particularly producing lengthy written responses • Writes slowly • Writes illegibly	• Allow students to report answers orally • Have students work in pairs or cooperative groups with a designated recorder • Extend the amount of time for task • Allow students to use a computer or portable keyboard such as an Alpha Smart to type their answers • Allow students to record answers on a tape recorder
Create a graph, table, chart, and so on	• Does not draw these forms adequately • Draws slowly • Cannot draw straight lines and position them correctly • Cannot write numbers small enough for small grids on graph paper	• Provide templates for forms, either blank or partially filled-in • Use larger grids • Provide finger grip or nonskid rulers to give better control • Allow students to use a spreadsheet program that creates the template
Align numbers	• Misaligns numbers when performing written calculations	• Provide paper with vertical lines or place lined paper sideways • Allow students to use graph paper
Use manipulatives	• Has difficulty moving, making patterns with, building, or aligning concrete manipulatives	• Provide a range of sizes and shapes of manipulatives • Have students work in pairs or cooperative groups with a person designated to move manipulatives • Use nonskid matting on desks to prevent sliding • Use computer software that emulates manipulatives, if available
Cut	• Has difficulty using scissors • Unable to cut accurately • Cuts slowly	• Provide precut pieces • Have students work in pairs with one student cutting
Copy information or problems from the text or board	• Writes slowly • Misaligns numbers • Copies inaccurately	• Provide handouts to minimize copying • Reduce the number of problems to be copied